2018-2019 Junior Division: Problems 1 solutions

J1. Early on a very hot day, a greengrocer places 20 kilograms of courgettes on display outside his shop. At that moment, the courgettes are 99% water. It turns out to be the hottest day of the year, and as a result, the courgettes dry out a bit. At the end of the day, the greengrocer has not sold a single courgette, and the courgettes are only 98% water. What weight of courgettes does he have at the end of the day?

Solution

In the morning, the 20 kilograms of courgettes are 99% water. So the non-water part of the courgettes has a mass of 1% of 20 kilograms which is 0.2 kilograms.

At the end of the day, the courgettes are 98% water. The remaining 2% is still the 0.2 kilograms of non-water material (which does not change when the water evaporates). If 2% equals 0.2 kilograms, then 100% equals 50×0.2 kilograms which is 10 kilograms.

So, at the end of the day, the greengrocer has 10 kilograms of courgettes left,

J2. On a coastline there are three lighthouses.

The first light shines for 3 seconds, then is off for 3 seconds.

The second light shines for 4 seconds, then is off for 4 seconds.

The third light shines for 5 seconds, then is off for 5 seconds.

All three lights have just come on together.

When is the first time that all three lights will be off?

When is the next time that all three lights will come on at the same moment?

Solution

The first light will be off between 3 and 6 seconds.

The second light will be off between 4 and 8 seconds.

The third light will be off between 5 and 10 seconds.

So all three will first be off after 5 seconds.

The first light comes on at multiples of 6 seconds.

The second light comes on at multiples of 8 seconds.

The third light comes on at multiples of 10 seconds.

The smallest multiple of 6, 8 and 10 is $6 \times 4 \times 5 = 120$.

So all lights next come on together after 120 seconds or 2 minutes.

J3. Two people are jogging back and forth on a straight road between two places which are 11 miles apart. One jogs at 5 miles per hour and the other at 6 miles per hour. They set off from opposite ends of the road at the same time. Determine where they are when they meet for the second time and how long it has taken them.

Solution

Let the ends of the road be *A* and *B*. Let jogger 1 (J1) start from *A* jogging at 6 mph and jogger 2 (J2) from *B* at 5 mph. Their approach speed is 11 mph so they meet for the first time after 1 hour.

J1 is then 5 miles from *B* and J2 is 6 miles from *A*.

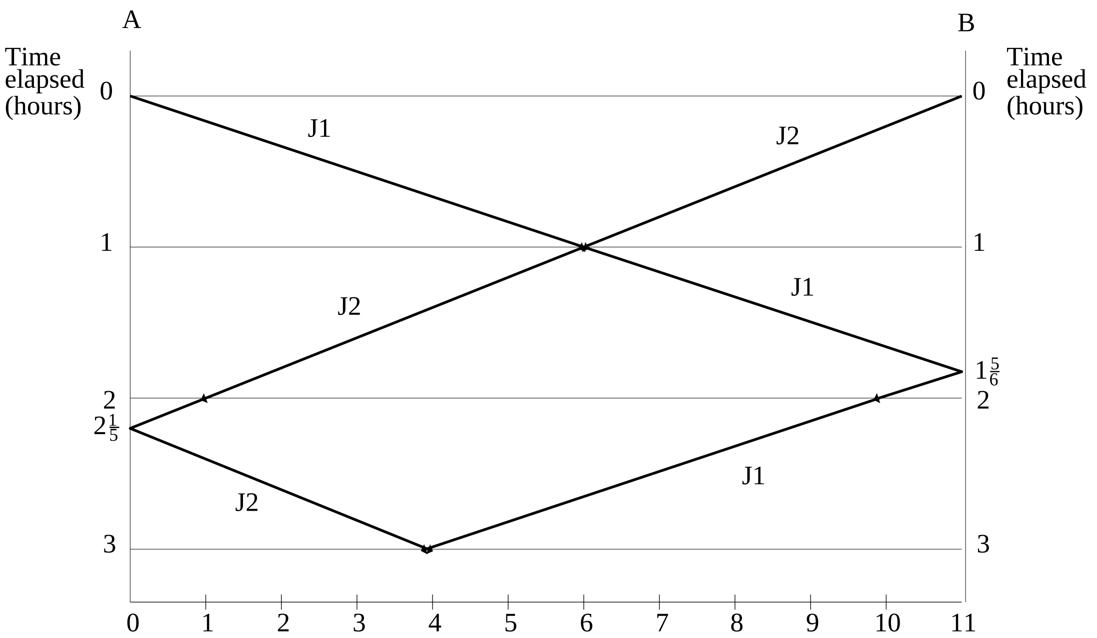
Thus J2 reaches A after a further $6 \div 5 = 1.2$ hours.

In that time J1 has jogged $6 \times 1.2 = 7.2$ miles and so has reached *B* and is 2.2 miles from *B* and jogging towards *A*.

This means that the joggers are $11 - 2.2 = 8.8$ miles apart and their approach speed is still 11 mph. So they meet $8.8 \div 11 = 0.8$ hours later.

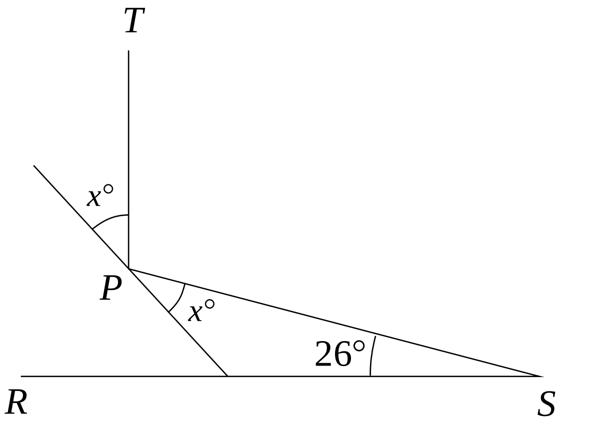
Thus the total length of time is then $1 + 1.2 + 0.8 = 3$ hours. In this time jogger 1 has jogged 18 miles so the meeting point is 7 miles from point *B*.

This time chart might be of assistance:



The joggers meet for the second time after 3 hours, at the point 7 miles from where the faster jogger set off (or 4 miles from where the slower jogger set off).

J4. A beam of light shines from point S, reflects off a reflector at point P, and reaches point T so that PT is perpendicular to RS and $\angle RSP = 26°$ as shown below. Find angle $x°$.



Solution 1

In the diagram, extend TP to meet RS at A. Since AT is perpendicular to RS

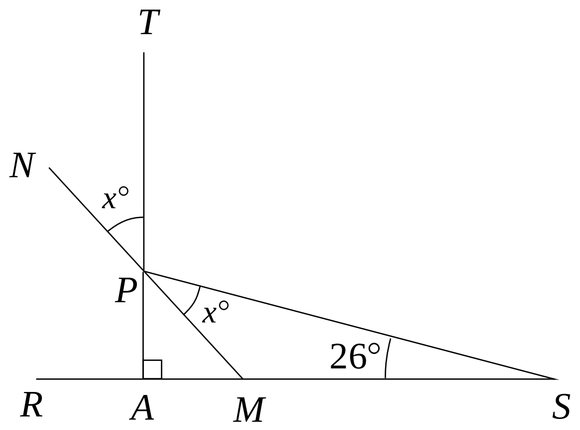
$$\angle SPA = 180° - 90° - 26° = 64°.$$

Label points M and N. Since $\angle TPN$ and $\angle MPA$ are vertically opposite angles, they are equal, so

$$\angle MPA = x°.$$

Since $\angle SPA = 2x° = 64°$, $x = 32$.

Thus, the angle $x°$ is 32°.



Solution 2

In the diagram, draw the line PU parallel to RS.

$$\angle SPU = \angle RSP = 26° \text{ (alternate angles)}$$

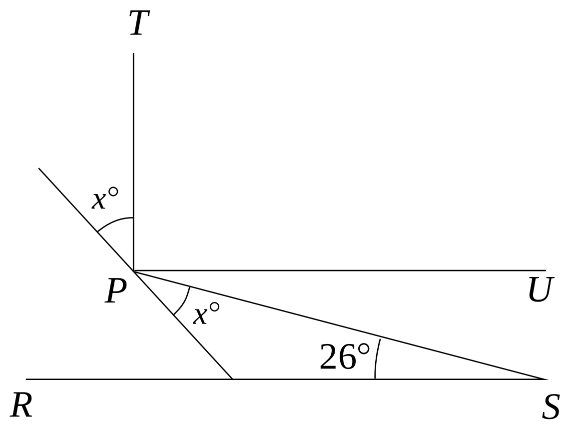
As PT is perpendicular to RS it is also perpendicular to PU so $\angle TPU = 90°$.

Thus, at the point P, we have

$$x° + 90° + 26° + x° = 180°$$

$$2x° = 180° - 90° - 26° = 64°.$$

Thus, the angle $x°$ is 32°.



J5. Emma started with a rectangle of paper. With one straight cut she divided it into a rectangle and a square. She took the rectangle and with one straight cut divided it into a rectangle and a square, which was smaller than the previous one. She kept repeating this process until eventually the final rectangle was a square with sides 1 centimetre and she was left with a pile of squares of paper. The average area of the squares was a two digit number of square centimetres.

What were the dimensions of the original rectangle?

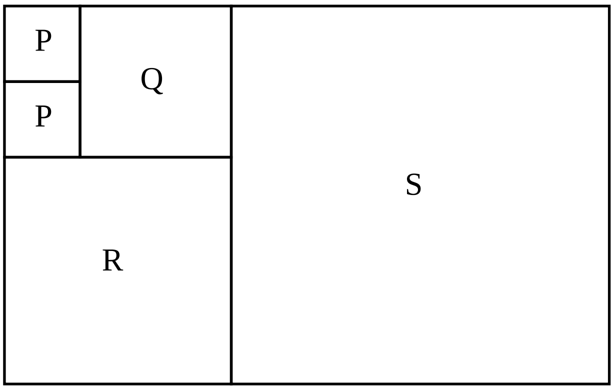
Solution

This solution reverses the dissection until the dimensions of the original paper are found.

Start with the two 1 cm squares (P) and put them together to make a rectangle with longer side 2 cm.
Add a 2 cm square (Q) to make the next rectangle with longer side 3 cm.

Add a 3 cm square (R) to make the next rectangle with longer side 5 cm.

Add a 5 cm square (S) to make the next rectangle with longer side 8 cm and so on.



Now analyse the dimensions. (Note that the sides of each square form a Fibonacci sequence where each term is the sum of the previous two terms.)

square side	area of largest square	total area	number of squares	average area
1	1	1	1	1
1	1	2	2	1
2	4	6	3	2
3	9	15	4	not an integer
5	25	40	5	8
8	64	104	6	not an integer
13	169	273	7	39
21	441	714	8	not an integer
34	1156	1860	9	over 100

So the original piece of paper was 21 cm by 13 cm.