Junior Division 2017-2018 Round 1 Solutions

J1. In our local town we still have a grocer, newsagent, butcher and baker and they have shops next to each other. They are all members of the local golf club and their names are Alan, Bill, Colin and David (not, perhaps, in this order).

Colin and David shave themselves whereas the baker prefers to go to the barber's across the way. David and Alan often play golf with the newsagent and the baker. David's shop is next to the butcher's.

One of the men has a beard. What is his occupation?

Solution - one of many variations

'Colin and David shave themselves whereas the baker prefers to go to the barber's across the way.' Neither Colin nor David is the baker.

'David and Alan often play golf with the newsagent and the baker.' David and Alan are not the newsagent or baker.

Bill must be the baker and Colin must be the newsagent.

'David's shop is next to the butcher's.' David cannot be the butcher so he must be the grocer leaving Alan as the butcher.

The man with the beard is the butcher.

J2. Jonas travels 26 miles in 3 hours, partly on foot and partly by bike. He walks at 4 mph and cycles at 12mph. For what length of time does Jonas walk? How far does he cycle?

Solution 1

Let Jonas walk x miles and so he cycles $(26 - x)$ miles.

$$\frac{x}{4} + \frac{26 - x}{12} = 3$$

$$3x + 26 - x = 36$$

$$2x = 10$$

$$x = 5$$

So Jonas walks 5 miles in 1 hour 15 minutes and cycles 21 miles in 1 hour 45 minutes.

Solution 2

Let Jonas walk t hours and so he cycles $(3 - t)$ hours.

$$4t + 12(3 - t) = 26$$

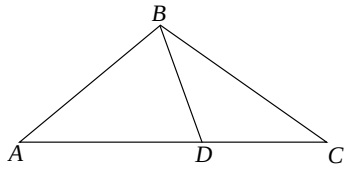
$$4t + 36 - 12t = 26$$

$$36 - 26 = 8t$$

$$8t = 10 \Rightarrow t = 1\frac{1}{4}$$

So Jonas walks for 1 hour 15 minutes and covers 5 miles and cycles for 1 hour 45 minutes and covers 21 miles.

J3. In the diagram $AB = BC = AD$ and $\angle BAD = 20°$.
Find $\angle CBD$.



Solution

$\angle BCA = \angle BAC = 20°$	(base angle of isosceles triangle *ABC*)
$\angle ABC = 180° - 2 \times 20° = 140°$	(third angle in triangle *ABC*)
$\angle ABD = \angle ADB$	(base angles of isosceles triangle *ABD*)
$\angle ABD + \angle ADB = 180° - \angle BAD$	(angle sum of triangle *ABD*)

Hence $\angle ABD = 80°$.
So $\angle CBD = \angle ABC - \angle ABD = 60°$.

J4. Two fractions are equally spaced between $\frac{1}{4}$ and $\frac{2}{3}$ on a number line. What are the two fractions?

Solution

Distance between $\frac{1}{4}$ and $\frac{2}{3}$ is $\frac{2}{3} - \frac{1}{4} = \frac{8}{12} - \frac{3}{12} = \frac{5}{12}$
One third of this distance is $\frac{5}{36}$.
So the first unknown fraction is $\frac{1}{4} + \frac{5}{36} = \frac{9}{36} + \frac{5}{36} = \frac{14}{36} = \frac{7}{18}$
and the second unknown fraction is $\frac{14}{36} + \frac{5}{36} = \frac{19}{36}$.
(Check: $\frac{19}{36} + \frac{5}{36} = \frac{24}{36} = \frac{2}{3}$ as required.)

J5. The value of 50! is the product of all the whole numbers from 1 to 50 inclusive, i.e.

$$50! = 1 \times 2 \times 3 \times 4 \times \ldots \times 49 \times 50.$$

Find how many times 2 will divide 50!.

Solution
There are 25 even numbers which are less than or equal to 50 so 25 factors are divisible by 2.
12 of these factors are divisible by 4 so divisible by a second factor of 2
6 of these factors are divisible by 8 so divisible by a third factor of 2
3 of these factors are divisible by 16 so divisible by a fourth factor of 2
Finally, 1 of these factors is divisible by 32 so divisible by a fifth factor of 2
There are no factors are divisible by 64 or more.
So 2 will divide 50! $25 + 12 + 6 + 3 + 1 = 47$ times.