Junior Division: 2016-2017 Round 2 Solutions

J1. Colin and Tom are on a camping holiday and, at their campsite, they make friends with Fiona. They ask her when her birthday is but, being a bit of a joker, Fiona tells them only that it is one of the following dates.

May 14,	May 15,	May 18,	June 16,	June 19,
July 12,	July 15,	August 12,	August 14,	August 16.

She then tells Colin the month of her birthday, but not the day in the month, whilst she tells Tom the day in the month, but not the month.

Immediately, Colin declares “Well, Tom certainly cannot know for sure when Fiona's birthday is” to which Tom replies “Ah, but now I do.” “And now I know when it is as well,” comes back Colin.

When is Fiona's birthday? Explain your reasoning.

Solution

If Fiona gave Tom day 18 or 19, then he would know immediately the exact date of the birthday (May 18 or June 19) as these are the only occurrences of 18 and 19. Since Colin is certain that Tom cannot be sure of the exact date, Tom concludes that Fiona must have given the month July or August to Colin.

As Tom now knows the full date exactly, Colin concludes that this must be one of July 15, August 14 or August 16 (it cannot be July 12 or August 12 as, knowing only the day in the month, Tom wouldn't be sure which of these it would be). Since Colin knows the month and says that he now knows for certain the exact date, this must be July 15 as, if Fiona had told him August, then both the dates August 14 and August 16 would still be possible.

Thus Fiona's birthday is July 15.

J2. The pages of George's book are numbered from 1. The page numbers have a total of 555 digits.

How many pages does the book have?

How many of the digits are a 5?

Solution

Pages 1-9 contain 9 digits.

Pages 10-99 contain $2 \times 90 = 180$ digits

This leaves $555 - 189 = 366$ digits at 3 digits per page.

So there are a further $366 \div 3 = 122$ pages.

So the book has $99 + 122 = 221$ pages.

Every 10th page ends in a 5, so 22 page numbers end in a 5.

Pages 50-59 and 150-159 have 5 as the tens digit, so a further 20 pages have tens digit 5.

Thus $22 + 20 = 42$ of the digits are a 5.

J3. Given

$$\begin{array}{ccc} & a & b \\ + & c & d \\ \hline e & f & g \\ \hline \end{array} \qquad \begin{array}{ccc} & g & f \\ + & e & d \\ \hline c & b & a \\ \hline \end{array}$$

where a, b, c, d, e, f, g are all single digits not necessarily all different. Find the values of a, b, c, d, e, f and g which makes **both** sums correct.

Solution

The maximum total of two 2-digit numbers is 198 so the letters $e = c = 1$

Then observe that a and g must be 8 or 9 in order to give a 3-digit answer

If $a = 8$ then $b + d > 10$, so $f = 0$ but then $f + d = 18$ which is not possible.

$$\begin{array}{ccc} & 8 & b \\ & 1 & d \\ \hline 1 & f & g \\ \hline \end{array} \qquad \begin{array}{ccc} & g & f \\ & 1 & d \\ \hline 1 & b & 8 \\ \hline \end{array}$$

If $a = 9$ and $f = 1$ then from the second sum $d = 8$, $g = 9$ and $b = 0$ but this does not work for the first sum ($8+0 \neq 9$)

$$\begin{array}{ccc} & 9 & b \\ & 1 & d \\ \hline 1 & 1 & g \\ \hline \end{array} \qquad \begin{array}{ccc} & g & 1 \\ & 1 & d \\ \hline 1 & b & 9 \\ \hline \end{array}$$

If $a = 9$ and $f = 0$ then the solution is

$$\begin{array}{ccc} & 9 & 0 \\ & 1 & 9 \\ \hline 1 & 0 & 9 \\ \hline \end{array} \qquad \begin{array}{ccc} & 9 & 0 \\ & 1 & 9 \\ \hline 1 & 0 & 9 \\ \hline \end{array}$$

Guidelines

J4. Peter is walking through a train tunnel when he hears a train approaching. He knows that on this section of track trains travel at 60 mph. The tunnel has equally spaced marker posts, with post 0 at one end and post 12 at the other end. Peter is by post 7 when he hears the train. He quickly works out that whether he runs to the nearer end or the further end of the tunnel as fast as he can (at constant speed) he will just exit the tunnel before the train reaches him.

How fast can Peter run?

Solution

Let the length of the tunnel be T miles.

As Peter is at post 7, the distances to the ends of the tunnel are $\frac{7}{12}T$ and $\frac{5}{12}T$ miles.

The train reaches the start of the tunnel (at post 12) at the time it would take Peter to run to the nearer end, i.e. the time to run a distance of $\frac{5}{12}T$ miles.

So if instead he runs to the further end (which is at post 0) he will reach $\frac{10}{12}T$ miles when the train is just entering the tunnel.

So he will have to run the remaining $\frac{2}{12}T = \frac{1}{6}T$ miles of the tunnel in the time the train takes to pass through the whole length of the tunnel at 60 mph, i.e. $\frac{T}{60}$ hours.

So Peter's speed is $\frac{\frac{T}{6}}{\frac{T}{60}} = \frac{60}{6} = 10$ mph.

J5. An old fashioned tram starts from the station with a certain number of men and women on board.

At the first stop, a third of the women get out and their places are taken by men. At the next stop, a third of the men get out and their places are taken by women. There are now two more women than men and as many men as there originally were women.

How many men and women were there on board at the start?

Solution

	Number of men	Number of women
From the station	m	w
After the first stop	$m + \frac{1}{3}w$	$\frac{2}{3}w$
After the second stop	$\frac{2}{3}m + \frac{2}{9}w$	$\frac{7}{9}w + \frac{1}{3}m$

There are now as many men as there were women originally:

$$\frac{2}{3}m + \frac{2}{9}w = w$$
$$\frac{2}{3}m = \frac{7}{9}w$$
$$6m = 7w$$

And there are two more women than men:

$$\frac{7}{9}w + \frac{1}{3}m = \frac{2}{3}m + \frac{2}{9}w + 2$$
$$\frac{5}{9}w = \frac{1}{3}m + 2$$
$$5w = 3m + 18$$
$$10w = 6m + 36$$
$$10w = 7w + 36$$
$$3w = 36$$
$$w = 12$$
$$m = 14$$

So there were 14 men and 12 women on the bus at the start.