Junior Division 2016-2017 Round 1 Solutions

J1. A school has fewer than 200 pupils.
When they line up in rows of 4 there is 1 extra pupil.
When they line up in rows of 5 there are 2 extra pupils.
When they line up in rows of 6 there are 3 extra pupils.

How many pupils could there be in the school?

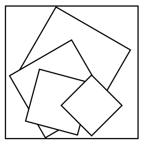
Solution 1
Rows of 4: 5, 9, 13, 17, …
Rows of 5: 17 pupils will leave 2 over
Rows of 6: for there to be 3 people left over the number must be divisible by 3 and be odd.
So 17 works for rows of 4 and 5 but not 6.
But if we add $5 \times 4 (= 20)$, it will still work for rows of 4 and 5.
So we get 37, 57, … and 57 is divisible by 3.
Thus 57 is one possible answer.
{Other possibilities are obtained by adding the lowest common multiple of 4, 5 and 6, i.e. 60.
So the possible numbers of members are 57, 117 and 177.}

Alternative solution:
(i) Rows of 5: Multiples of 5 all end in 5 or 0 so the situation with rows of 5 with 2 left over gives a total ending in 7 or 2.
(ii) Rows of 6: for there to be 3 left over the total must be odd and a multiple of 3.
So from (i) the total must and in 7: 7, 17, 27, 37, 47, 57,....., 187, 197.

Check for those that are multiples of 3: 27, 57, 87, 117, 147, 177.

(iii) Rows of 4: for there to be 1 person left over the total has to be 57, 117 or 177.

J2. Professor A. M. Nesia has a safe with a combination lock. In her journal, the note she uses to help her remember is this diagram →



and the year of her birth, 1941,
This reminds her that the code is a sequence of five perfect squares (square numbers) in ascending order where the mean = 19, median = 4 and mode = 1.
Find the combination.
Solution
With five numbers written in numerical order, the third one must be the median: _, _, 4, _, _
The mode is 1 and this has to be before the 4 so both numbers less than 4 must be 1: 1, 1, 4, _, _

Let the last two values be a and b.
The mean is 19 so the total = $1 + 1 + 4 + a + b = 19 \times 5$, i.e. $a + b = 89$.

We need two perfect squares which add up to 89:

Square numbers: 1, 4, 9, 16, 25, 36, 49, 64, 81 and to get a last digit of 9 it has to be 4 and 5 ie $a = 25$ and $b = 64$.

So the combination is 1142564.

J3. My petrol tank was a quarter full when I pulled into the petrol station. I put in £22.50 worth of petrol and noticed that the tank was now two thirds full. The cost was £1.20 per litre.

What is the capacity of the petrol tank?

Solution

Let a full tank cost £ x.

Fuel put into tank is $\frac{2}{3} - \frac{1}{4}\left(= \frac{5}{12}\right)$ of the capacity of the tank.

So,

$$\frac{5}{12}x = 22.50$$

$$x = (22.50 \times 12) \div 5 = 54$$

If a full tank costs £54 to fill and a litre costs £1.20 then the number of litres in a full tank is

$$54 \div 1.20 = 45.$$

The capacity of the petrol tank is 45 litres.

J4. A victorious football team in an open-top bus is scheduled to leave the home ground and arrive at the town hall at 11 am. If the bus travels at 15 mph it will arrive 8 minutes early. However if it travels at 10 mph it will arrive 8 minutes late. At what speed must it travel to arrive at 11 am exactly?

Solution

Let the distance be d miles and the required travel time t hours. Then

$$\frac{d}{15} = t - \frac{8}{60}$$

$$\frac{d}{10} = t + \frac{8}{60}$$

Adding

$$\frac{d}{15} + \frac{d}{10} = 2t$$

$$\frac{d}{12} = t$$

So the required speed is 12 mph.

J5. (a) Adam has a five-digit number

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When he places a 1 at the end of this number it becomes a six-digit number three times as large as the number he obtained when he places a 1 at the start.

Find the five-digit number.

(b) If you added a 1 in the same way to a 3-digit number how many times as large would it have to be?

Solution

(a)

$$* * * * * 1 = 3 \times 1 * * * * *$$

Let the five digit number be x.

$$10x + 1 = 3(100\,000 + x)$$

$$10x + 1 = 300\,000 + 3x$$

$$7x = 299\,999$$

$$x = 42\,857$$

(b)

Three digit number

$$* * * 1 = n \times 1 * * *$$

Let the three digit number be y.

$$10y + 1 = n(1000 + y)$$

$$(10 - n)y = 1000n - 1$$

List the possibilities for $n = 1$ to 9 and the only ones which give y as an integer are $n = 1$, 7 and 9 but $n = 1$ means that the value has not changed. In this case $10y + 1 = 1000 + y$, $9y = 999$. So $y = 111$.

However, $n = 7$ or 9 both lead to y as a four-digit number (2333 or 8999).

So this only works for a three-digit number when the number is 1 times as large i.e. unchanged.