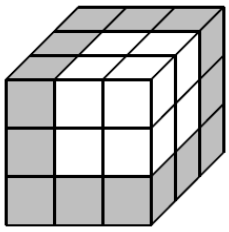
2015-2016 Junior Solutions Round 1

J1. The solid large cube in the diagram is made up from individual smaller cubes. The individual small cubes were all originally white until some faces were painted grey as shown. What is the largest possible number of completely white cubes?



Solution

There are $3 \times 3 \times 3 = 27$ small cubes in all.

12 cubes with at least one grey face are visible.

So the remaining 15 small cubes could be all white.

So the largest possible number of all white cubes is 15.

J2. Tessa was practising subtraction. After doing a calculation correctly, she copied it into her notebook but was distracted and instead of writing down each figure, she wrote either the figure one higher or the figure one lower in the sequence 0, 1, ..., 9. What she wrote down is shown alongside.

Find the correct figures, explaining how you worked it out.

$$\begin{array}{r} 2910 \\ -1497 \\ \hline 2106 \end{array}$$

Solution

In the last column, the 0 should be a 1; the 7 could be a 6 or an 8 and the 6 a 5 or a 7. We need two figures which add to 11 so the 7 should be a 6 and the 6 a five..... etc to give

$$\begin{array}{r} 3801 \\ 2586 \\ \hline 1215 \end{array}$$

J3. Robin was born in May when his mother (whose birthday fell in July) was 37 and his fath birthday fell in August) was 40. They decided to have a special party if any of them ever birthday on which all three had prime number ages. Show that however long they live, th only be one special party.

Solution

Robin will never have a special party, because on his birthday his parents' ages differ by of them will always be even so not prime.

Father will never have one, because on his birthday the parents' ages will again differ by

Mother has a chance of a special party whenever neither her age nor Father's is a multiple that is, when she is 38, 41, 44, ... etc, Father is 41, 43, 46, ... etc, and Robin is 0, 3, 6, ... e only prime number for Robin is 3 itself.

Thus the only possibility is Mother's 41st, when Robin is 3 and Father is 43. These are al this is the unique special party.

Guidelines

J4. On July 1st, Bill started to read a book recommended by friends, and, by reading the sam of pages each day of the month, managed to finish it on the 31st.

Another friend, Clare, also started reading the book at the same time. She read a quarter number of pages on the first of the month and, on each following day, one more page tha previous day. She also finished on the 31st.

How many pages did the book contain?

Solution

Let x be the number of pages read each day by Bill.

$$31x = \frac{x}{4} + \left(\frac{x}{4} + 1\right) + \left(\frac{x}{4} + 2\right) + \left(\frac{x}{4} + 3\right) + \ \ldots \ + \left(\frac{x}{4} + 30\right)$$

$$\text{so} \quad 31x = 31 \times \frac{x}{4} + 1 + 2 + \ \ldots \ + 30$$

$$\text{so} \quad 31x = 31 \times \frac{x}{4} + 465$$

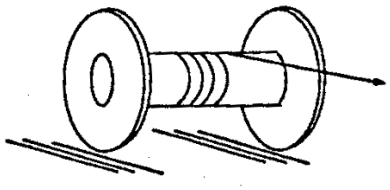
$$\text{so} \quad x = \frac{x}{4} + 15$$

$$\text{so} \quad \frac{3}{4}x = 15$$

$$\text{so} \quad x = 20.$$

Answer: the number of pages is $20 \times 31 = 620$.

J5.



An almost empty bobbin is pulled along a flat sur thread which is wrapped around it, as shown in th The diameter of the inner reel is 5 cm and that of wheel is 10 cm. Assuming no slipping or sliding, has the bobbin moved when the end of the thread moved 12 cm?

Solution

In one revolution the bobbin will move 10π cm on the flat surface.

This will unwind an additional 5π cm of thread.

So the free end of the thread will move 15π cm.

Thus the bobbin moves 2/3 of the distance moved by the end of the thread.

i.e. When the end of the thread moves 12 cm the bobbin will move 8 cm.

(Try it!)