2013-2014 Junior Solutions Round 2

J1.

The date of the second Thursday of a particular month is a square number. What is the date of the last Wednesday of that month?

Explain your reasoning.

Solution

The second Thursday is between the 8th and 14th so the square number is 9.

Then 9th, 16th, 23rd, 30th are Thursdays.

The last Wednesday is the 29th except in February in non-leap years when it is the 22nd.

J2.

One family outing last summer included an impromptu sports day with five events in which 4, 2 and 1 points were awarded for the first three places in each event. Douglas and John tied with 8 points each. Jackie came next with 7, and Bill and Colin each had 6.

Colin didn't win any event but gained points in three of them. He beat both Bill and Douglas in the 200 metres but was well behind John in the High Jump.

Jackie won the Long Jump, but was well out of the points in the High Jump.

Bill was the only child who gained points in every event, his best effort being in the 100 metres.

Who were the first three children in the 400 metres event and in what order did they finish?

Explain your reasoning.

Solution

Jackie won the Long Jump, but came nowhere near a place in the High Jump: 4 in the LJ and 0 in the HJ.

Bill was the only child who gained points in every event, his best effort being in the 100 metres: the only way to get a total of 6 points would be to get 1,1,1,1,2 and he got 2 in the 100 metres.

Colin didn't win any event but was 'placed' in three of them: to get a total of 6 from three events he has 0,0,2,2,2. Bill got 2 in the 100 metres so Colin must be 0.

Colin beat both Bill and Douglas in the 200 metres: so must have got 2 points leaving Douglas with 0.

	200 metres	High Jump	Long Jump	100 metres	400 metres	Total
Douglas	0					8
John						8
Jackie		0	4			7
Bill	1	1	1	2	1	6
Colin	2			0		6

If we look at the 200 metres then John or Jackie must have won but Jackie's total is only 7 so it must have been John. This means for Jackie to have a total of 7 means getting 1 and 2 points for the 100 metres and 400 metres respectively. This means that Colin must have got 0 for the 400 metres leaving him with 2 points in the HJ and the LJ.

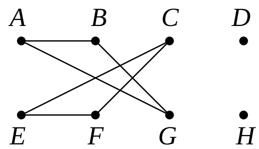
Colin was well behind John in the High Jump: John must have won the High Jump. The rest of John's events must be 0. So Douglas must have won the 100 metres and 400 metres.

In the 400 metres race Douglas, Jackie and Bill came in that order.

	200 metres	High Jump	Long Jump	100 metres	400 metres	Total
Douglas	0	0	0	4	4	8
John	4	4	0	0	0	8
Jackie	0	0	4	1	2	7
Bill	1	1	1	2	1	6
Colin	2	2	2	0	0	6

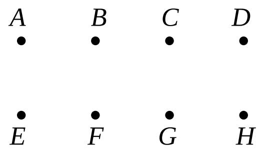
J3.

Triangles are called ‘congruent’ when they are identical. This means they are the same size and shape, although they can be in different positions. For example, triangles *ABG* and *EFC* are congruent.



Non-congruent triangles must be different is some respect.

How many **non-congruent** triangles can be formed by joining the dots on the grid below?



Explain your reasoning.

Solution

3 with base length 1, for example *AEF*, *CEF* and *DEF*. *BEF* is congruent to *AEF*.

3 with base length 2, for example *AEG*, *BEG* and *DEG*. *CEG* is congruent to *AEG*.

2 with base length 3, for example *AEH* and *BEH*. *AEH* and *DEH* are congruent as are *BEH* and *CEH*.

Total 8 different triangles.

J4.

A block of four postage stamps, with perforations along the joins so that they can be easily separated, have values in pence as shown:

1	2
4	3

Show that it is possible to make every postage value from 1p to 10p using either a single stamp or a number of stamps joined along lines of perforations.

Using a different set of stamp values in the block of four, it is possible to make every postage value from 1p to a higher limit than 10p. Construct an arrangement of stamp values which reaches the highest possible limit.

Are there any other solutions which give this limit? Explain.

Solution

1p, 2p, 3p and 4p are single stamps.

5p: 1p and 4p
6p: 1p and 2p and 3p
7p: 4p and 3p
8p: 1p and 4p and 3p
9p: 2p and 3p and 4p
10p: 1p and 2p and 3p and 4p

To find the maximum possible total, first count the number of combinations of stamps available:

4 single stamps	4 joined pairs of stamps	4 joined threes	all 4 stamps
4	4	4	1

so there is a total of 13 combinations, so highest possible total value is 13p, with each value being obtained in exactly one way.

But can we arrange the stamp values to achieve this?

There must be a 1p stamp, otherwise we could not have this value.

If there were another 1p stamp, then there would be 2 possible ways to get 1p and another value would be missed. So there is only one 1p stamp.

The next lowest value stamp must then be 2p.

Case 1: Put the 2p stamp next to the 1p stamp:

1	2

This gives a total of 3p from these 2 joined stamps, so the next lowest value stamp must be 4p, and the final stamp has value $13 - 1 - 2 - 4 = 6$ pence. There can be only one way to get 6p value, and so the 4p stamp cannot be next to the 2p stamp, making the final arrangement:

1	2
4	6

Does this arrangement work? 5p is obtained from 1p and 4p joined, and 7p up to 12p are what is left when 1p to 6p are taken from 13p. Finally 13p is obtained from all four stamps. So yes.

Case 2: Put the 2p stamp diagonally opposite the 1p stamp. The next stamp value must then be 3p, and the final stamp value $13 - 1 - 2 - 3 = 7$ pence. The positions are then

1	3
7	2

Finally check that all values can actually be obtained from joined stamps:

1p, 2p, 3p, 7p single stamps,	4p: 1p and 3p
5p: 2p and 3p	13p: all four stamps

and the remaining values are left behind when others are detached.

There are no further different possible positions for the 2p stamp (it must be either next to or diagonally opposite the 1p stamp) and so we have found the only two sets of stamp values possible.

J5.

A brother and sister, Peter and Fiona, are always thinking about numbers.

On his birthday Peter said “My age is a square number.”

His older sister Fiona said “That's right, but the sum of our ages and the difference of our ages also give squared numbers.”

Peter replied “In three years time, both our ages will be prime numbers.”

Fiona replied “Three years ago, both our ages were also prime numbers.”

What are the ages of Peter and Fiona now?

Solution

(We assume throughout that Peter and Fiona are not aged well over a hundred.)

Peter's current age is a square number but in three years, it will be a prime so his current age must be even.

Possible ages	4	16	36	64	100
In 3 years time	7	23	39	67	103
3 years before	1	13	33	61	97

So Peter is either 16 or 64 or 100.

In three year's time, Fiona's age will be prime and therefore odd so at present it is even. Let it be x, and if Peter is 16 then $x - 16$ and $x + 16$ are both square numbers. So we need two square numbers which differ by 32.

Square numbers	1	**4**	9	16	25	36	**49**	64	81	100
+ 32	33	**36**	41	48	57	68	**81**	96	113	132

So from this, if Peter is 16 then Fiona is 20 or 65 (impossible because odd).

But if Peter is 64 then $x - 64$ and $x + 64$ are both square numbers. So we need two square numbers which differ by 128.

Square numbers	1	4	9	**16**	25	36	49	64	81
+ 128	129	132	137	**144**	163	164	177	192	209

So from this, if Peter is 64 then Fiona is 80. But three years ago, Fiona would have been 77 which is not prime. So Peter is not 64.

But if Peter is 100 then $x - 100$ and $x + 100$ are both square numbers. So we need two square numbers which differ by 200.

Square numbers	1	4	9
+ 200	201	204	209

So from this, if Peter were 100 there are no pairs of squares so Peter is not 100.