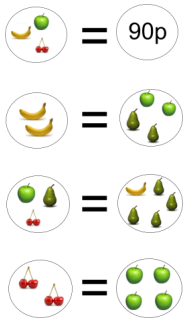
# 2012-2013 Junior Solutions Round 1

## J1

A mathematical greengrocer had the display shown alongside in his shop window. The poster shows how the costs of one apple, one banana, one pear and a pair of cherries are related.
How much does each kind of fruit cost?




*Solution*

1. Non-algebraic

From picture 4, a pair of cherries cost the same as 2 apples.

Using this with picture 3, 3 apples and a pear cost the same as a banana and 4 pears. So 3 apples cost the same as a banana as 3 pears,

But from picture 2, a banana costs the same as an apple and a pear so the previous statement becomes 3 apples cost the same as an apple and 4 pears or 2 apples costs the same as 4 pears so 1 apple costs the same as 2 pears.

So a pair of cherries costs the same as 4 pears and a banana costs the same as 3 pears.

We can now interpret picture 1 and obtain the cost of 9 pears is 90p and hence 1 pear costs 10p, a pair of cherries cost 40p, a banana costs 30p and an apple costs 20p.

2. Algebraic

$$b + c + a = 90 \quad (1)$$
$$b = a + p \quad (2)$$
$$a + c = b + 3p \quad (3)$$
$$c = 2a \quad (4)$$

Eliminating $c$

$$b + 3a = 90 \quad (5)$$
$$b = a + p \quad (6)$$
$$3a = b + 3p \quad (7)$$

Eliminating $b$

$$4a + p = 90$$
$$2a = 4p \quad (9)$$

Hence

$$9p = 90$$

So the costs are:

Pear = 10p, Apple = 20p, Banana = 30p, A pair of Cherries = 40p

**J2.**

Alice, Bill, Colin, Daisy and Edward play a game in which each is a frog or a toad. A frog's statements is always false and a toad's statements is always true.

(a) Alice says that Bill is a toad.
(b) Colin says that Daisy is a frog.
(c) Edward says that Alice is not a frog.
(d) Bill says that Colin is not a toad.
(e) Daisy says that Edward and Alice are different kinds of animals.

Who of Alice, Bill, Colin, Daisy and Edward are the frogs?

*Solution*

Label the statements A - E.
First, assume Alice is a toad. This means Bill is a toad.
And hence from D, Colin is a frog.
So B is not true, so Daisy is a toad.
Thus E is true, and, since Alice is a toad, Edward is a frog.
Finally C is false, so Alice is a frog.
But this contradicts the original assumption that Alice is a toad which must therefore be wrong.

So actually Alice is a frog. Then A gives that Bill is a frog.
And hence D is false, so Colin is a toad.
Then B gives Daisy is a frog.
And E is false, so Edward is a frog.
Finally C is false, so Alice is a frog.
This matches with the initial statement that Alice is a frog, and so we have the solution.

Alice, Bill, Daisy and Edward are all frogs.

## J3.

Two shops, Jessies and Laskops, sell the same model of camera, Jessies for £16 more than Laskops. But then Jessies introduced a special offer giving 25% off, making the camera £2 cheaper than from Laskops.
What is the price of the camera from each shop, taking the special offer into account?

*Solution*
Let £$J$ be the original price from Jessies and £$L$ be the price from Laskops.
Then

$$J = L + 16$$

and

$$\tfrac{3}{4}J = L - 2.$$

So

$$\tfrac{1}{4}J = 18,\ J = 72 \text{ and } L = 56.$$

Hence the special offer price from Jessies is £54 and the price from Laskops is £56.

## J4.

Gran, Helen, Mum and Dad sat down one evening to play a card game each starting with 20 counters. As a general rule Gran has a fair amount of luck but, on this occasion it deserted her completely and she finished well down.
Mum gained half of what Gran had lost and Helen was six down. Dad finished with four times as many counters as Gran.
How many counters did Mum finish with?

*Solution*
*At the end, let x be the number of counters that Gran lost. Then the numbers each player had were:*

| | |
|---|---|
| Gran | $20 - x$ |
| Helen | 14 |
| Mum | $20 + \tfrac{1}{2}x$ |
| Dad | $4(20 - x)$ |

Total :

$$\begin{aligned} 54 - \tfrac{1}{2}x + 80 - 4x &= 80 \\ 4\tfrac{1}{2}x &= 54 \\ x &= 12 \end{aligned}$$

So Mum finished with 26.

**J5.**

In mathematics, the notation 11! is short for $11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$.
Similarly, another example is, $4! = 4 \times 3 \times 2 \times 1$ and so $4! = 24$.

What is the greatest factor of 11! that is one greater than a multiple of 6?

*Note: 11! is spoken of as 'eleven factorial'.*

*Solution*

Mutiples of 6 are even and also divisible by 3 so the answer is an odd number which has no factor of 3.

The odd factors of 11! which are not divisible by 3 are 5, 7 and 11 but, because $10 = 5 \times 2$ there is an extra factor of 5.

So the required number is a factor of $5 \times 7 \times 5 \times 11 = 1925$. This fails as 1924 is not a multiple of 3.

The next biggest factor is $5 \times 7 \times 11 = 385 = 94 \times 6 + 1$.

The answer is 385.