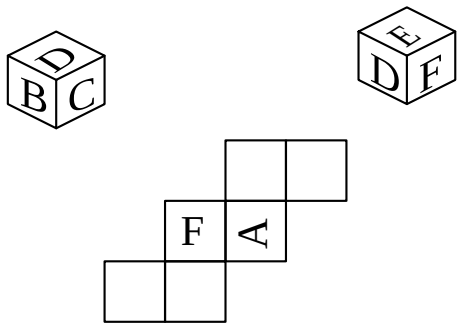
2011 Junior Set 1 solutions

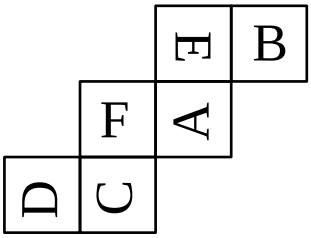
J1.

A cube has the letters A, B, C, D, E and F marked on the faces. Below are two views of the cube. Use these to complete the net. **Describe each of your steps.**



Solution

Looking at the net shown and the die on the right; since A is to the right of F, A is opposite D.
Comparing the two dice, B is opposite F. This fixes the positions of A, B, D, E and F and so C must be opposite E.
Check the orientation of the letters.
Construct the net as shown.



J2. A four-digit PIN number '*rrss*', where *r* and *s* are different digits, is a multiple of 15. Which values of *r* are not possible?

Solution

Since the number is a multiple of 15 then it is a multiple of 5 and 3.
Being a multiple of 5 means that *s* must be 5 or 0.
Assuming *r* cannot be zero.

rrss is a multiple of 3 so, by the standard digit-sum result, $2r + 2s$ must also be a multiple of 3 and therefore $r + s$ is a multiple of 3.

Consider $s = 5$: $r + 5$ is a multiple of 3 which leaves $r = 1, 4, 7$

Consider $s = 0$: r must be a multiple of 3 which leaves $r = 3, 6, 9$

The values of *r* which are not possible are 2, 5 and 8.

Note - this interpretation should be allowed

If *r* is allowed to be zero, in the case where $s = 0$, *r* must be a multiple of 3 giving $r = 0, 3, 6, 9$. But *r* cannot be 0 as that would mean *r* and *s* would be the same.
So the values of *r* which are not possible are 0, 2, 5 and 8.

J3. At a meeting of the Young Historical Society (YHS) in a local library the number of children present was three times the number of parents. At this meeting, each boy borrowed 12 books from the library, each girl borrowed 17 books and each parent borrowed 9 books. In total 305 books were borrowed. How many girls were there at the YHS meeting?

Solution

Let the number of boys, girls and parents be b, g and p respectively.

$$\begin{aligned} p &= \tfrac{1}{3}(b + g) \\ 12b + 17g + 9p &= 305 \\ 12b + 17g + 9 \times \tfrac{1}{3}(b + g) &= 305 \\ 15b + 20g &= 305 \\ 3b + 4g &= 61 \end{aligned}$$

Given that b and g are positive integers this gives a number of solutions

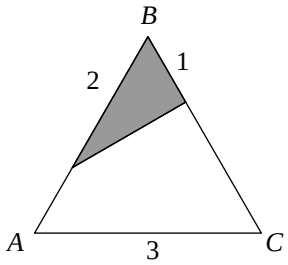
(b, g) : (3, 13), (7, 10), (11, 7), (15, 4), (19, 1).

However, p is also a positive integer which means that $b = 11$ and $g = 7$ is the only solution.

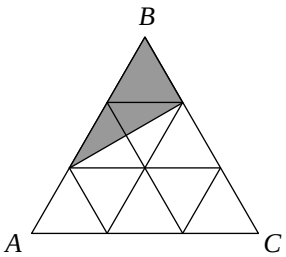
The number of girls is 7.

J4. The triangle ABC is equilateral. What fraction of it is shaded?

Explain your reasoning.



Solution



We can split the equilateral triangle into 9 equilateral triangles with side length 1.

The two small shaded triangles cover half of a rhombus formed from 2 triangles so their area is 1.

The total shaded area is the same as 2 triangles.

The area of the shaded part would be $\frac{2}{9}$ of the large triangle.

J5. Six cousins have a get together and discover that the mean of their ages is 19. Four of the ages are 21, 19, 23 and 11 and range of all six ages is 20. Find the other two ages. **Explain your reasoning.**

Solution

There are 6 cousins. Since their mean is 19 the total of their ages is 6×19 which is 114.

The total $21 + 19 + 23 + 11 = 74$.

Let the missing numbers be a and b. From above it follows that $a + b = 40$.

Since the range is 20 if 23 is the largest then the smallest must be 3 which means that the other missing number must be 37. But 37 would then be the largest not 23.

If 11 was the smallest then the largest would be 31 which means the other missing number is 9. However, this would then be the smallest not 11.

This means that a and b must be the largest and smallest numbers in the set giving:

$$a + b = 40$$

$$a - b = 20$$

Therefore the missing numbers are 30 and 10.