The Scottish Mathematical Council

www.scot-maths.co.uk

MATHEMATICAL CHALLENGE 2011−2012

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, Professor L E Fraenkel,
The London Mathematical Society and The Scottish International Education Trust.
The Scottish Mathematical Council is indebted to the above for their generous support and gratefully acknowledges financial and other assistance from schools, universities and education authorities.
Particular thanks are due to the Universities of Aberdeen, Edinburgh, Glasgow, Heriot Watt, Stirling, Strathclyde, and to Preston Lodge High School, Bearsden Academy, Beaconhurst School and Northfield Academy.

Junior Division: Problems 2

J1. A water-tank can be filled by any combination of three different taps. With the smallest tap the tank can be filled in 20 minutes. With the middle tap the tank can be filled in 12 minutes. With the largest tap the tank can be filled in 5 minutes. How long does it take to fill the tank with all three taps running?

Explain your reasoning.

J2. The children were playing with the bouncing balls in the playground when I arrived there. A ball reached my full height before falling back down and I am 1.6 metres tall. “Wow! That was some bounce!”, I said, but the boy responded, “That was nothing − you should have seen the first bounce. That was the third bounce”. Each bounce of this particular ball is 20% less than its previous bounce. What height was the first bounce?

Explain your reasoning.

J3. Five schoolchildren are weighed in pairs. The readings (in kilograms) are:

58, 60, 61, 64, 64, 65, 67, 68, 70 and 71.

Find the weights of the individual children.

J4. A set of cards, numbered from 1 to 19, are placed face down on a table. Nine players each pick up two cards. The remaining card is then turned over. The player who achieves the highest total with their two cards plus the number on the remaining card is the winner.

Is it possible for all nine players to have the same total?

If so, what can this total be?

Explain your reasoning.

J5. A computer whizz claims that his program has found some numbers which satisfy Fermat's equation $x^n + y^n = z^n$ for a large integer n.

He tells his 10 year old brother that

$x = 31415926536$ $y = 89173261421$ $z = 90354441655$

Almost immediately his brother says that there cannot be any value of n which will work for these numbers. The computer whizz checks his program and finds a bug.

How did his brother know there was a bug?

END OF PROBLEM SET 2