2010 Junior Set 1 solutions

J1. A lottery win is shared between three people. Allan gets 20 percent more than Jane, and 25 percent more than Charlie. Jane's share is £3,600. How much money does Charlie receive?

Solution 1

Jane gets £3600 and Allan gets 20% more than her. So Allan gets
£3600 + £720 = £4320.

Allan gets 25% more than Charlie so Charlie gets $\frac{4}{5}$ of Allan's amount.

$\frac{1}{5}$ of £4320 is £864. So Charlie gets £4320 − £864 = £3456.

Solution 2

Let the amounts be £a, £j and £c.
Immediately, $j = 3600$.

$$a = j \times 1.2 = 4320$$

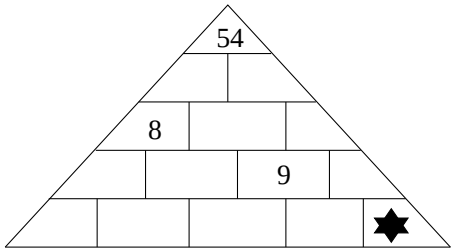
$$c \times 1.25 = a = 4320$$

$$c = \frac{4}{5} \times 4320 = 3456$$

Charlie gets £3456.

J2. In each region of the triangle shown there is a whole number, three of which are given.

Each number is the sum of the two numbers immediately below it and all numbers are different. Find out which number must be in the region marked with the star and explain why.



Solution

As the numbers are all different, the smallest possible numbers in the bottom row are 1, 2, 3, 4 and 5. Thus the smallest possible number in the second row is 3.

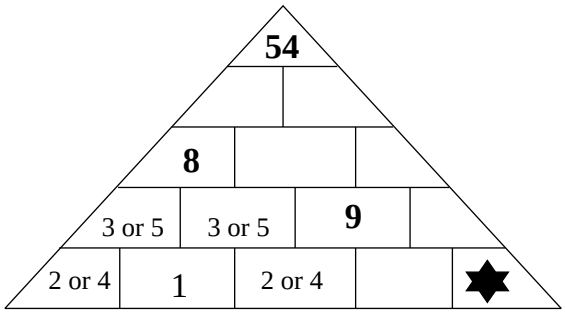
To get the 8, the cells below it must contain 3 and 5.

Below the 3, we must have 1 and 2.

Below the 5, we could have 2 and 3 or 1 and 4, but 3 cannot be repeated so 5 comes from 1 + 4.

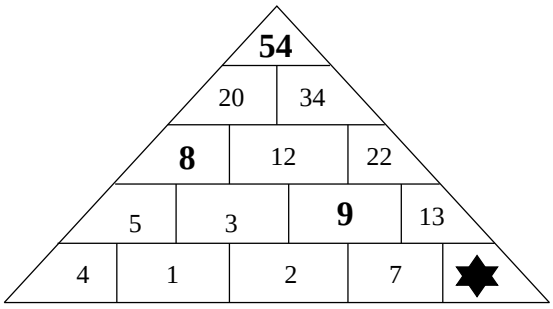
As 1 feeds into 3 and into 5, it must be directly below the 8 with 4 and 2 on either side.

So we have:



But 4 cannot feed into 9 as 5 is already used. So 4 is on the left.

This means that below 8 we have 5 and 3 (rather than 3 and 5).



The rest can now be filled in: 12, 20, 34, 13, 7 and finally the required number is 6.

J3. Amanda, Brian and Claire enter the school talent contest. They each perform in one of three rooms in the morning and in a different one of the three rooms in the afternoon.

We know that

- Amanda's act is maths magic,
- one pupil moves from the hall to the gym,
- Claire is in the drama studio after lunch,
- Brian's morning room is taken by the singer in the afternoon,
- one pupil's act is juggling.

Find out where each person performs in the morning and in the afternoon, and what their act is.

Justify your answer.

Solution

The initial information is:

	Morning	Afternoon	Act
Amanda			Maths Magic
Brian			
Claire		Drama Studio	

Brian's morning room is taken by the singer in the afternoon tells us that he is not the singer so Claire must be the singer and Brian must be the juggler. It also tells us that Brian was in the Drama Studio in the morning.

	Morning	Afternoon	Act
Amanda			Maths Magic
Brian	Drama Studio		Juggler
Claire		Drama Studio	Singer

Amanda must move from the hall to the gym. So we now know that Claire was in the gym in the morning and Brian was in the hall in the afternoon.

	Morning	Afternoon	Act
Amanda	Hall	Gym	Maths Magic
Brian	Drama Studio	Hall	Juggler
Claire	Gym	Drama Studio	Singer

J4. Write down any whole number containing four digits. Now write down a second number containing the same digits in a different order. Show that, when you take the smaller number from the larger number, you obtain a multiple of 9.
Explain why this always works for any four-digit number.

Solution
Numerical example: 1234; reverse 4321
Difference = $3087 = 9 \times 343$

Consider any four digits a, b, c and d and form the two numbers

$$abcd \quad \text{and} \quad cadb.$$

Written out, the numbers are

$$1000a + 100b + 10c + d \quad \text{and} \quad 1000c + 100a + 10d + b.$$

Subtracting these numbers gives $900a + 99b - 990c - 9d$ which is divisible by 9.
No matter how the digits are rearranged, the same pattern will occur.

J5. The areas of the faces of a cuboid are 12 cm^2, 16 cm^2 and 48 cm^2. Find the volume of the cuboid.

Solution
Let the lengths edges be x cm, y cm and z cm where $x \leqslant y \leqslant z$.
Then $xy = 12$, $xz = 16$ and $yz = 48$.
Multiplying these results gives:

$$(xy) \times (xz) \times (yz) = 12 \times 16 \times 48$$

$$x^2y^2z^2 = 9216.$$

But the volume is given by xyz and

$$xyz = \sqrt{9216} = 96.$$

Hence the volume of the cuboid is 96 cm^3.