The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2010−2011

Entries must be the unaided efforts of individual pupils.
Solutions must include explanations and answers without explanation will be given no credit.
Do not feel that you must hand in answers to all the questions.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE
The Edinburgh Mathematical Society, Professor L E Fraenkel,
The London Mathematical Society and The Scottish International Education Trust.

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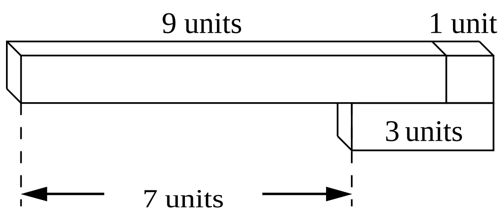
Junior Division: Problems 2

J1. You are given three rods of lengths 1, 3 and 9 units. Using these rods, you could measure 7 units as shown.

Show how you could measure each whole number length up to 13 units.



By adding a fourth rod, it is possible to measure all whole number lengths up to 40 units. What is the length of this extra rod?

Explain your answer.

J2. 

You have three boxes, each containing two *identically* wrapped Easter eggs.

One box contains two milk chocolate eggs (M), one contains two plain chocolate eggs (P) and the third contains one *milk chocolate* egg and one *plain chocolate* egg. The boxes are labelled MM, PP or MP according to their contents.

However, someone has switched all the labels so that every box is now incorrectly labelled.
You are allowed to take out one egg at a time *from* any box, check what type it is and put it back. By doing this you can correctly label all three boxes.
What is the smallest number of eggs you would need to check in order to label the boxes correctly?

Explain your answer.

J3. Take all the prime numbers between 30 and 60 and place them in a row in such a way that:

(a) the sum of the two largest numbers and the numbers between them in the row is 233;
(b) the sum of the smallest prime, the middle one in size and the numbers in between them in the row is 133;
(c) the difference between the first and last primes in the row is 6;
(d) the difference between the second and sixth primes in the row is also 6.

Justify your conclusion.

J4. Four cards, each numbered with a different whole number, are placed face down. Four people, Gavin, Jack, Katie and Luke, in turn select two of these cards, write down their total, and then replace the two cards. Gavin's total is 6, Jack's 9, Katie's 12 and Luke's 15.

Two of the cards are then turned over and their total is 11.

Determine the numbers on each of the cards.

J5. An old-fashioned rectangular billiard table has only four pockets, one at each corner. The lengths of the sides of the table form a whole number ratio.

Show that, if the ratio is 5 : 2 and a ball is hit from one corner at an angle of 45°, it will land in a pocket after 5 rebounds.

If the ratio of the sides were $m : n$, where m and n are different whole numbers, with no common factor, and the ball were hit from a corner at an angle of 45°, show that the ball would always drop into a pocket after a number of rebounds. How many rebounds would there be in this case?

END OF PROBLEM SET 2