2009 Junior Set 2 solutions

J1. In Wuppertal Zoo, there is a tank containing 41 spectacular tiger fish. Each male fish has 111 stripes while each female has only 37 stripes. Unfortunately the male fish caught a disease and two thirds of them died. How many stripes were on display in the tank after this?

Solution

Note that 111 is three times 37 so each male fish had three times as many stripes as a female fish. When two thirds of the male fish die, two thirds of the male stripes disappear. This gives the same number of stripes on male fish as we would have if **all the male fish survived** but had lost two thirds of their stripes and so had only 37 stripes instead of 111.

So the number of stripes now on display is $41 \times 37 = 1517$.

J2. A lion would take four hours to eat one sheep; a leopard would take five hours; and a bear would take six hours. Assuming that the lion, the leopard and the bear were friends, how long would they take to devour a single sheep?

Solution

Let the time taken be t hours, then

$$\frac{1}{4}t + \frac{1}{5}t + \frac{1}{6}t = 1 \qquad (\times 60)$$

$$15t + 12t + 10t = 60$$

$$t = \frac{60}{37} = 1\frac{23}{37}$$

The eating time is $1\frac{23}{37}$ hours.

J3. Corinne and Elspeth live in different houses in a street with 12 houses. Every day, Corinne receives more letters than anyone else in the street and Elspeth receives fewer letters than anyone else in the street. One day there were 56 letters delivered to the street. What is the smallest that the difference between the number of letters that Corinne received and the number that Elspeth received, can be?

Solution

Since Corinne received at least one more than anyone else and Elspeth received at least one less than everyone else, and there were more than two recipients, the difference must be at least 2.

If the difference was exactly 2, then everyone, apart from Corinne and Elspeth, received the same number of letters. But this is equivalent to all 12 households getting the same number of letters (as Corinne could give Elspeth one letter). But, since 56 is not a multiple of 12, it is not possible for each house to get the same number so the difference cannot be 2.

Checking to see if the difference could be 3, suppose that Elspeth receives 0, Corrinne 3, and, of the remaining 10 houses, n get 1 letter and $(10 - n)$ get 2 letters; then $0 + n + 2(10 - n) + 3 = 56$, but this gives $n < 0$.

Similarly, if Elspeth receives either 1 letter or 2 letters, $n < 0$.

However there is a positive value for n, when Elspeth receives 3 letters and Corrinne receives 6 letters; in that case, $3 + 4n + 5(10 - n) + 6 = 56$, with solution $n = 3$; in this case, 3 neighbours receive 4 letters and 7 receive 5 letters.

So the smallest difference is 3.

J4. One evening a psychology lecturer invited some students to her house. She had a bowl of cherries which she had carefully counted so that there was the same number for each student. As she passed round the bowl of cherries, the first student took one more cherry than his share. The second student took one more cherry than the first student and the third student took one more cherry than the second student. At this point, with half the cherries eaten, the lecturer took the bowl and divided the remainder equally among the remaining students, each receiving one less cherry than she originally intended. How many cherries were there?

Solution

The first three students took 6 extra cherries. Since the remaining students each got 1 less than they should have done, there must have been six students remaining. Thus there were a total of nine students.

Let x be the number of cherries each student should have received. Since half the cherries were gone after the first three students, the greedy students took their fair share and six more whilst the remaining six each had $x - 1$ cherries. Hence

$$3x + 6 = 6(x - 1) = 6x - 6$$

$$12 = 3x \Rightarrow x = 4$$

Each fair share is 4 cherries. Thus the total number of cherries was 36.

J5. How many squares, of *all* sizes, are seen on a standard, 9×9 Sudoku grid?

Solution

Consider the numbers of squares of all possible sizes:

1×1 squares	there are $9 \times 9 = 81$
2×2	these could have their top left corner in any square which is not in the last row or column giving $8 \times 8 = 64$ possibilities
3×3	$7 \times 7 = 49$

etc

Hence the number of squares is $1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 + 81 = 285$.