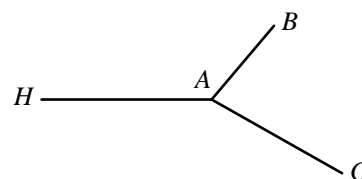


2009 Junior Set 1 solutions

- J1** From his home H , a travelling salesman has to make 12 deliveries, 4 to each of the towns A , B and C as shown on the diagram.



For his travel he gets a mileage allowance of 50p per mile. He claims for 4 return journeys from home to each of the towns A , B and C . But he actually made the deliveries on a total of 4 trips, each trip going from H to A to B to A to C to A and back to H . If, by his dishonest claim, he made an additional £160, how far is it from H to A ?

Solution

In his actual 4 trips, he travelled the road from A to B and back 4 times, the road from A to C and back 4 times and the road from H to A and back 4 times.

In the 4 journeys to each of A , B and C that he claimed for, he travelled the road from A to B and back 4 times, the road from A to C and back 4 times and the road from H to A and back 12 times. So he claimed for extra mileage equal to 8 return journeys from H to A which is 16 times the distance from H to A . Since he got an additional £160, the distance from H to A earned him £10. At 50p per mile that distance must be 20 miles.

- J2** Four old-timers, John, Willie, Andy and Greig, take their wives to a tea-dance. At one point during the dance, the partners were as follows:

- Flo is dancing with Willie.
- Hettie is dancing with Beeb's husband
- Mary is dancing with Hettie's husband.
- Andy is dancing with Greig's wife.
- Greig is dancing with Willie's wife.

Identify the married couples.

Solution

Since Flo is dancing with Willie, by the next two statements, Willie cannot be Beeb's husband (as Hettie is dancing with him) and also cannot be Hettie's husband (as Mary is dancing with him). Willie cannot be married to Flo, since by the last statement Greig is dancing with Willie's wife. So Willie has to be married to Mary.

As Greig is dancing with Mary, by the third statement, Greig is Hettie's husband. So Andy is dancing with Hettie and by the second statement Andy is married to Beeb. That leaves John to be married to Flo. Thus:

Willie	and	Mary	are married
Greig	and	Hettie	are married
Andy	and	Beeb	are married
John	and	Flo	are married

J3 Four boys, each of whom work at the same rate, decided to lay a path from a cottage to the beach. Because of an argument, two of the boys only worked for one day and the job was then completed by the other two. This resulted in the work taking two more days than originally intended. How long would it have taken if all four had worked to lay the path?

Solution 1

If 4 boys completed the project, then, on day 1, they would complete from A to B, and after 2 days be at C, after 3 days at D etc.



If 4 boys worked on day 1, but only two thereafter, it would take 2 days to complete from B to C, and C to D etc; that is, after 3 days they would be at C, after 5 days at D etc.

Since the project took 2 extra days, the path stretched from A to D and the time taken by the 4 boys would have been 3 days.

Solution 2

Consider the amount of work done by a boy in a day to be 1 unit and let the scheduled length of the work be x days. Since the work was to be done by 4 boys it follows that the total amount of work was $4x$ units.

However, 2 boys just did 1 unit each and the other 2 did $x + 2$ units each. Adding these we have:

$$\begin{aligned} 1 + 1 + (x + 2) + (x + 2) &= 4x \\ \Rightarrow 6 + 2x &= 4x \\ \Rightarrow 2x &= 6. \end{aligned}$$

So the planned time required by the 4 boys was 3 days.

Solution 3

Let the work done by 1 boy in 1 day be x and suppose that the job should have taken y days. It actually took $y + 2$ days.

Then, work done by 4 boys in y days =

Work done by 4 boys in 1 day + work done by 2 boys in $(y + 1)$ days

$$4xy = 4x + 2x(y + 1).$$

Hence, $2y = 2 + y + 1$, giving $y = 3$.

Time required by 4 boys is 3 days.

J4 Mr and Mrs McLeod have six children – Andrea, John, Eilidh, Rory, Fiona and Pat. Just before Christmas, the six children went on their own to town to do some shopping. They all spent some money, each spending a whole number of pounds. When they returned home, they told their parents, in a roundabout way, how much they had spent as follows: Andrea and John together had spent £26, Eilidh and Rory together had spent £20 and Fiona had spent £9. They didn't say how much Pat had spent, but they did say that one of them had spent £15 more than the average for all the children. Mr and Mrs McLeod thought about this and then started arguing about how much Pat had spent. Explain why they were arguing and say what you can about the amount Pat had spent.

Solution

The total amount of money we are told about is £55 and so the average expenditure is greater than £9. It has to be a whole number since one child spent £15 more than the average and each child spent a whole number of pounds.

So if the child who spent £15 more than the average was not Pat it could only have been either Andrea or John, with one of them spending £25 and the other £1. In that case, the average must have been £10 and the total spent was £60. Thus Pat spent £5.

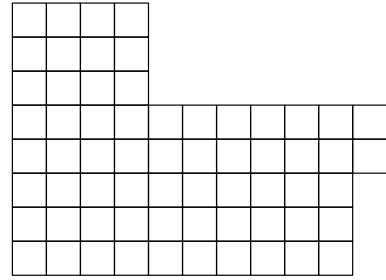
On the other hand, if it was Pat who spent £15 more than the average, suppose he spent £ x .

Then $x = 15 + \frac{55 + x}{6}$. Thus $x = 29$, i.e. Pat spent £29.

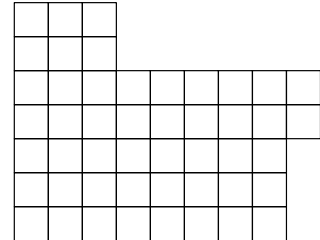
They argued because there are two possible solutions to this problem.

J5

By cutting along the lines, can you divide the shape on the right into two pieces which can be fitted together to make an eight by eight square?



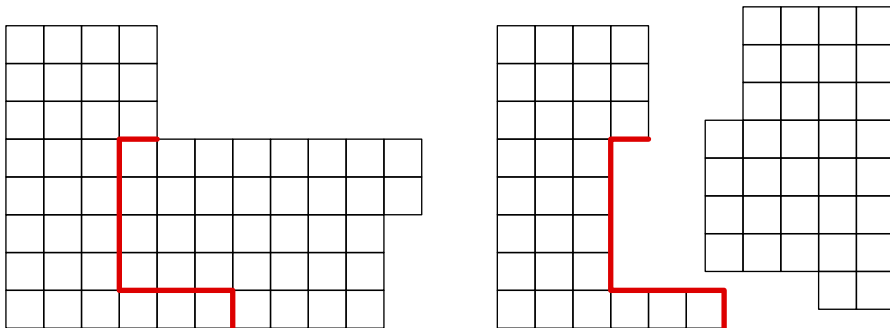
By cutting along the lines, can you divide the shape on the right into two pieces which can be fitted together to make a seven by seven square?



Justify your conclusions.

Solution

There are 64 squares in the first diagram so the task may be possible. Note that there are 8 squares along the left hand side of the shape and seven on the longer horizontal edge. So we need to cut in 1 square to make an edge of 8 squares. Since there are 4 squares on the short top edge we need to cut down 4 squares as that edge will fit along the top to make 8 squares. Finally since the right hand edge is now to be the bottom and it has only 2 squares at the bottom level we need to cut out a further 6 edges as shown.



The second shape cannot be cut into two pieces to make a seven by seven square as there are only 48 squares in the shape and we need 49.