

2008 Junior Set 1 solutions

- J1.** After taking part in a diving competition, and before the results were announced, the five girls who had taken part were discussing how they thought the competition had gone.
- Alice said: "Beth was first; Deb was last."
Beth said: "I was second; Alice was third."
Claire said: "I was third; Deb was fourth."
Deb said: "Beth was third; Alice was fourth."
Emma said: "I was first; Claire was last."
- When the results were announced, there were no ties and it turned out that each girl had made one true statement and one false statement. Find the placings of the five girls and explain your reasoning.

Solution

Since one of Beth's statements is true, either Beth was second or Alice was third. So Beth was not third.

Since one of Deb's statements is true, and Beth was not third, Alice must have been fourth.

Since one of Claire's statements is true and Alice was fourth, Claire must have been third.

Since one of Beth's statements is true and Claire was third, Beth must have been second.

Since one of Emma's statements is true and Claire was third, Emma must have been first.

That leaves Deb to be last.

So the order was 1. Emma 2. Beth 3. Claire 4. Alice 5. Deb.

It is then straightforward to check that then each made one true and one false statement.

- J2.** On a 26-question test, 8 points were credited for each correct answer and 5 points were deducted for each wrong answer. If all questions were answered, how many were correct if the score was zero?

If Fred and Bernie both scored more than zero, but Fred scored 10 times as many points as Bernie, how many did Fred score correctly and how many did Bernie score correctly? (Again assume that all questions were answered).

Solution 1.

If x answers were correct and y were wrong, then $x + y = 26$ and $8x - 5y = 0$.

So $8x = 5(26 - x)$ i.e. $13x = 130$. So 10 questions are answered correctly for a score of zero.

Let Fred get x_1 correct and y_1 wrong and Bernie gets x_2 correct and y_2 wrong. We then have the following equations: $x_1 + y_1 = 26$, $x_2 + y_2 = 26$, $8x_1 - 5y_1 = 10(8x_2 - 5y_2)$.

This last equation shows that y_1 must be even and so x_1 must be even. It also shows that x_1 must be divisible by 5. Hence x_1 is a multiple of 10. By the first part, it is bigger than 10 and on a 26-question test can only be 20. Thus Fred scored 20 questions correctly giving a points total of 130 while Bernie scored 11 correctly giving a score of 13.

Alternative Solution without algebra:

Each time one more question is answered correctly, the score changes by 13 marks. If initially none are correct, the score is $-5 \times 26 = -130$. So 10 questions answered correctly are required to make the score 0.

The number of questions over 10 which are answered correctly gives a score which is a multiple of 13. But if Fred scored 10 times as many points as Bernie, he must have scored 130 points and Bernie 13 since the total number of questions is 26. So Fred got 20 questions correct and Bernie got 11.

- J3.** The Mathematical Challenge offices have a six-digit phone number. This is a special number because you can rearrange the six digits to make either three consecutive two-digit numbers or two consecutive three-digit numbers. The secretary knows this but has forgotten the number. The chairperson informed her it was the largest possible such number. What is the number the secretary had to remember?

Solution

Any such number must have at least three distinct digits since if it only had two there could not be three consecutive two-digit numbers.

The largest six-digit number with three distinct digits is 999987. This can be re-arranged as 997,998 and 97, 98, 99. So this is the largest six-digit number with the required properties.

- J4.** Travelling to a pop concert I know that if I average 30mph for the journey I will arrive 1 hour early, but if I average only 20mph for the journey I will arrive 1 hour late. How fast do I need to travel to arrive exactly on time?

Solution

Let the distance travelled be d miles.

The time taken at 30mph is $\frac{d}{30}$ hours and at 20mph is $\frac{d}{20}$ hours.

So

$$\begin{aligned}\frac{d}{20} - \frac{d}{30} &= 2 \\ 10d &= 1200 \\ d &= 120\end{aligned}$$

The times travelled at therefore 4 hours and 6 hours. So if the journey is to be 5 hours, the speed must be $\frac{120}{5} = 24$ mph.

- J5.** Sam had never fully understood the points system in football and felt that the scoring of goals should be encouraged. His idea is that 10 points should be awarded for a win, 5 points for a draw and 1 point for each goal scored, whatever the result of the match. Therefore even if you are losing 0-5 and have no hope of winning, a goal scored might make all the difference between promotion and relegation. This was tried with three teams, Hubs, Dins and Rungs. Each team scored at least one goal in every match and no team played another more than once. Hubs scored 8 points, Dins 14 points and Rungs scored 9 points. Find the score in each match.

Solution

The total number of points awarded is 31. The number of points available per match here is 12. So only two games were played.

Dins never loses as Hubs and Rungs never win.

If Dins plays twice, Rungs plays once so must draw with Dins at a score of 4 – 4. But then since Dins scores at least one goal against Hubs, their total would be at least 15. So it follows that Dins only plays once. This means that Hubs and Rungs play each other. Either Hubs or Rungs only plays once so the score between them must be 3 – 3 or 4 – 4. It cannot be 4 – 4 as this would give Hubs at least 9 points. So it must be 3 – 3 and Hubs plays only once. Thus Rungs then plays Dins and loses 1 – 4.