The Scottish Mathematical Council

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MATHEMATICAL CHALLENGE 2007−2008

Entries must be the unaided efforts of individual pupils. Solutions must include explanations.

Answers without explanation will be given no credit.

CURRENT AND RECENT SPONSORS OF MATHEMATICAL CHALLENGE ARE

The Edinburgh Mathematical Society, Professor L E Fraenkel,

The London Mathematical Society and The Scottish International Education Trust.

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Junior Division: Problems 1

J1. Three friends visit a museum and walk up a flight of stairs. Ross goes up one step at a time starting with his left foot on the first step. Sheila goes up two steps at a time starting with her left foot on the second step and Tom starts with his left foot on the third step and goes up three steps at a time.

Investigate these questions and explain your answers.

(a) Which is the first step that all three will tread on?

(b) Which is the first step that all three will tread on with their right foot?

(c) Which is the first step that all three will tread on with their left foot?

J2. A group of boys found a chestnut tree with the chestnuts just ready for picking. One of the boys climbed the tree and was able to knock down some chestnuts. He had just enough to give himself and the other boys three chestnuts each, with none left over. Then three of their friends joined them. They found that it was not possible to share the chestnuts evenly among the group.

However, when one more chestnut was picked, it was possible to give each boy two chestnuts, with none left over. How many boys were there altogether? **Explain your reasoning.**

J3. Show that there is no five-digit number which uses each of the digits 1,2,3,4,5 such that the numbers formed

by the first digit is divisible by 1,
by the first two digits is divisible by 2,
by the first three digits is divisible by 3,
by the first four digits is divisible by 4,
by the first five digits is divisible by 5.

Explain your reasoning.

J4. In this question, you are only allowed to shade complete squares.

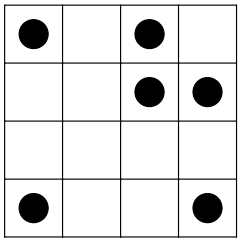
(a) In how many different ways is it possible to shade one half of this rectangle?

(b) In how many different ways is it possible to shade one third of this rectangle?

(c) In how many different ways is it possible to shade one quarter of this rectangle?

(d) In how many different ways is it possible to shade one fortieth of a rectangle made up of 80 squares?

J5. The diagram shows a 4 × 4 grid containing 6 black spots. These 6 spots are so placed that no three of them lie in a line, either horizontally, vertically or diagonally, but if you add one more spot, there will always be such a line of three spots.



What is the largest number of black spots you can place on such a 4 × 4 grid with this property i.e. no three spots are in a line but if you add any one spot there will always be a line of three?

What is the smallest number of black spots you can place on such a 4 × 4 grid with this property?

Explain your answers.

END OF PROBLEM SET 1