2006 Junior Set 2 solutions

J1. In cleaning out a drawer, Mrs Smith found two old watches which she and her husband had discarded. She wound them up, and, after setting them accurately, started both watches at the same time. An hour later she noticed that her old watch had gained one minute while her husband’s had lost two minutes. Checking them from time to time, it was clear that her old watch was running consistently fast and her husband’s consistently slow. Next morning, when she looked at the watches again, it was 7 o’clock on her old watch and 6 o’clock on her husband’s. What time was it when she started the watches running?

Solution

Every hour Mrs Smith’s watch gained 1 minute and her husband’s lost 2 minutes, so they differed by 3 minutes. So they would differ by an hour after a multiple of 20 hours.

Since it was the next day when she consulted the watches, that must have been after 20 hours. At that time her watch showed 7.00 a.m. but had gained 20 minutes. So the actual time was 6.40 a.m. So the watches were started 20 hours earlier at 10.40 a.m.

J2. A quiz had only 3-point questions and 5-point questions. The best possible score is 100 and there are 26 questions? How many of each type are there?

Caitlin and Zak each attempt the quiz. Caitlin scores one and a half times as many points as Zak. Every question that Zak answers correctly, Caitlin also answers correctly, but she answers an additional 10 questions correctly. What were the scores of Caitlin and Zak? (No partial marks were awarded.)

Solution

Since 3 × No. of 3-point questions plus 5 × No. of 5-point questions is 100, the No. of 3-point questions must be a multiple of 5. We also note that there cannot be more than 20 5-point questions and so there must be at least 6 3-point questions. Hence there must be at least 10 3-point questions. This gives the following table:

No. of 3-point Qs :	10	15	20	25
No. of 5-point Qs :	16	11	6	1
Total score :	110	100	90	80

Thus there are 15 3-point questions and 11 5-point questions.

Since Caitlin answers an additional 10 questions, these will give her an extra 30 to 50 points. But the number of additional points she gets is equal to half of Zak’s score and one third of her own score. By the last remark, these additional points must be less than 100/3 and so no more than 33. By answering 10 3-point questions she gets 30 points, by answering 9 3-point questions and 1 5-point question she gets 32 points, by answering 8 3-point questions and 2 5-point questions she gets 34 points which is too large. Thus she either answers 10 additional 3-point questions or she answers 9 additional 3-point questions and one 5-point question. The latter would mean that Zak’s total score was 64 and must come from a selection of at most 5 3-point questions and 10 5-point questions. Thus he can get at most 15 points from the 3-point questions and so needs another 64-15 = 49 points. So he needs all the remaining 10 5-points questions but then cannot achieve the remainder, 14, as an exact multiple of 3.

Thus Caitlin must answer 10 additional 3-point questions. In that case, Zak’s score is 60 and Caitlin’s is 90.

J3. The Broon family from Glebe Street, all eleven of them, went on a shopping spree to Auchenshuggle. Ten of them spent exactly £20 each but Horace was extravagant. He spent £10 more than the average for the whole family. How much did Horace spend?

Solution

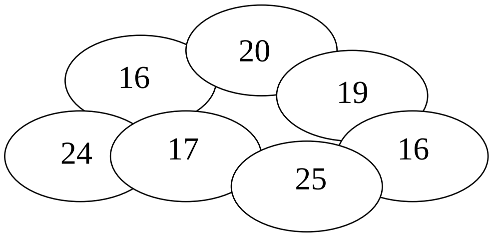
Suppose that Horace spent £x.

Then the total spent is $10 \times 20 + x = 200 + x$ so the average is $\frac{1}{11}(200 + x)$. Hence

$$\begin{aligned} x &= 10 + \frac{200 + x}{11} \\ 11x &= 110 + 200 + x \\ 10x &= 310 \end{aligned}$$

So solving gives $x = 31$.

J4. A merchant has an odd collection of barrels of wine and one barrel of beer as shown below with their capacities in gallons.



He gets rid of all of it by selling off some barrels of wine to one customer, twice that quantity of wine to another customer and keeping the barrel of beer for himself. How many gallons did the barrel of beer contain?

Solution

Since the wine is divided so that one customer gets twice as much as the other, the total number of gallons of wine is a multiple of 3.

Now the total number of gallons altogether is 16+20+19+24+17+25+16=137. So we need to remove one barrel and be left with a multiple of 3. Since 137 leaves a remainder 2 on division by 3, the possible ones to remove are 20 and 17.

If we remove 20, we get $117 = 3 \times 39$. So one customer would get 39 gallons of wine. But no combination of the remaining barrels of wine gives 39.

If we remove 17 we get $120 = 3 \times 40$. So one customer gets 40 = 16 + 24 gallons of wine and the barrel of beer contains 17 gallons.

J5. On a Wednesday in the middle of July in the Millennium year, Mrs Peat was exhibiting her collection of rare flowering plants to members of her horticultural society. She was particularly proud of three luxuriously flowering plants. They were quite delicate and each individual flower only lasted for a single day. The plant with crimson flowers blossomed every fourth day, the one with lavender flowers every seventh day and the one with yellow flowers every thirteenth day. The members of the horticultural society were delighted so Mrs Peat invited them to come again. “Let us meet again on the same Wednesday next year, which is the first time they will all be in bloom together again”. Was Mrs Peat’s statement correct?

Solution

A Wednesday in the middle of July in the following year would be 364 days after the Wednesday in the middle of July in the Millenium year (as no leap year intervened). Now 364 is divisible by 4, 7 and 13 and indeed is equal to $4 \times 7 \times 13$. So it is the smallest number which is divisible by all three. Thus Mrs Peat’s statement was correct.