2006 Junior Set 1 solutions

J1. During a very hard winter, Liam had only enough hay and corn to feed his six horses fo another 30 days and it would be another 75 days before spring would arrive. On the seventh day, before feeding time, Liam sold four of his horses. Will he be able to feed his remaining two horses for the rest of the winter? **Explain your reasoning.**

Solution

When Liam sold the horses, 69 days of winter remained and he had enough food to feed 6 horses for another 24 days. That is he could feed 2 horses for 72 days. The food will last out.

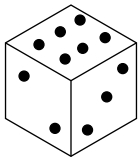
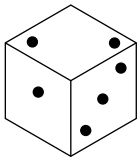
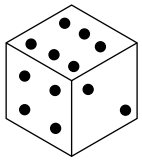
Alternative

At the start, Liam had $6 \times 30 = 180$ portions of horse food.

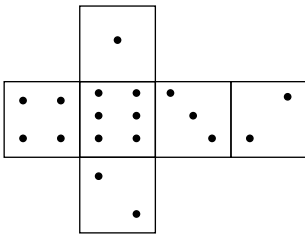
He used $6 \times 6 = 36$ portions whilst he still had 6 horses so had 144 portions left.

With 69 days to go and 2 horses to feed, he needs 138 portions so the food will last out.

J2. A dice maker makes mistakes when painting the spots on some dice. Below are three views of one of the dice he makes. How many spots are there on the bottom face in view 1 (i.e. the face opposite the six)? **Explain your reasoning.**

Solution



Observe that we can see faces with 1,2,3,4 and 6 spots.

In all three views we see a face with 2 spots. Place that face so that the spots run from top left to bottom right. Now look at the faces adjoining that face along its sides (left and right). We see faces with 3 spots, 1 spot and 4 spots. If there was only one face on the dice with 2 spots, there would be exactly 2 faces adjacent to it along its sides to left and right. So there must be two faces with 2 spots. The other faces will have 1, 3, 4 and 6 spots. We can now draw a plan of the cube and we see that the face opposite the six has 2 spots.

J3. Alice has three daughters all of whom are at least one year old. Alice challenged a friend Morag to work out the ages from the following clues:

- the sum of their ages is 11;
- the product of their ages is either 16 years less or 16 years more than Morag's age.

Morag said she could not identify the ages and would need another clue so Alice said that the daughter whose age in years is greatest is learning to play the clarinet.

Morag now knew the three ages. What were they, and how did Morag know?

Solution

From the first clue, the possible ages are

1, 1, 9;	1, 2, 8;	1, 3, 7;	1, 4, 6;	1, 5, 5;
2, 2, 7;	2, 3, 6;	2, 4, 5;	3, 3, 5;	3, 4, 4.

The products, p, of these ages are

9, 16, 21, 24, 25, 28, 36, 40, 45, 48.

Write down $p - 16$ and $p + 16$ which give 'possibilities' for Morag's age (obviously, not all are possible!):

$p - 16$:	−7, 0, 5, 8, 9, 12, 20, 24, 29, 32
$p + 16$:	25, 32, 37, 40, 41, 44, 52, 56, 61, 64

The possible age 32 occurs in both lists, but no other age does. If Morag takes any of the ages in the 2 lists other than 32, she would automatically know what the children's ages were from the corresponding product of children's ages. But if her age is 32, she can't tell between the children's ages whose products give 48 and 16. Morag must be 32 as she requests another clue; this final clue then excludes the case with children's ages 3, 4, 4.

J4. The islands of North Oost and South Oost have fewer than 2000 residents all together. North Oost is bigger having at least 50 more residents than South Oost. One third, one fourth, one fifth and one seventh of the population of North Oost are all whole numbers. The sum of these whole numbers gives the population of South Oost. What is the population of South Oost?

Solution

The population of North Oost is less than 2000 and is divisible by 3, 4, 5 and 7 and so is divisible by the product $3 \times 4 \times 5 \times 7 = 420$.

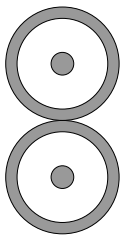
Hence the only possible values of the population of North Oost are 420, 840, 1260 and 1680.

The sum $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{7} = \frac{140 + 105 + 84 + 60}{420} = \frac{389}{420}$. Hence we can obtain the following table:

North Oost	420	840	1260
South Oost	389	778	1167
Difference	39	72	
Total		1618	2427

Hence the population of South Oost is 778.

J5. The diagram shows two wheels of the same diameter. The lower wheel is fixed and the upper wheel rotates without slipping about the lower wheel, the two wheels always being in contact. How many times does the upper wheel turn on its axis in making a complete revolution of the lower wheel?



Solution

Make a mark at the top of the upper wheel. On the journey round the lower wheel, this mark will be in contact with the lower wheel at the lowest point of the lower wheel when half the journey has been made. At this stage, the mark on the moving wheel is again at the top of it and so the moving wheel has made one complete revolution about its own axis. Thus there will be two complete revolutions for the whole journey.